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String Theory backgrounds with Torsion in the presence of Fermions and implications for Leptogenesis

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String Theory backgrounds with Torsion in the presence of Fermions and implications for Leptogenesis

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Abstract. Fermions couple in a universal way to the the Kalb-Ramond field which occurs in the gravitational multiplet of string theory. The Kalb-Ramond field is a source of torsion and can provide a background for fermion dynamics. Solutions for this background in terms of the string effective action are discussed and are fixed "points" of conformal invariance conditions. Non-perturbative fixed points for the torsion in the absence of matter are shown to be compatible with perturbative fixed points in the presence of matter. Fermion coupling to such backgrounds can lead to both CP and CPT violation. Hence torsion can give a new mechanism for leptogenesis. The presence of anomalies in the Bianchi identity for the torsion, in the presence of matter, is crucial in giving a background solution for the torsion and dilaton with an acceptable cosmology.

1. Introduction

Matter-antimatter asymmetry in primordial matter is a puzzle. The Standard Model (SM) of particle physics provides a partial answer. The answer uses the paradigm of Sakharov (1967) [1] which states sufficient conditions for a theory to produce baryogenesis (or leptogenesis):

- (i) Baryon number (B) violating interactions need to be present
- (ii) C (charge conjugation) and CP (where P denotes parity) are violated
- (iii) the theory is CPT invariant (where T is the time-reversal operator) and so in thermal equilibrium $\langle B \rangle = 0$. Hence, in order to have $\langle B \rangle \neq 0$ we need to be out of thermal equilibrium.

The violation of B in SM occurs through the triangle anomaly: the left and right chiral fields interact differently with the $SU(2)$ gauge fields of SM; C is broken e.g. in the neutrino sector; CP is broken by complex phases in the Yukawa couplings. In this way the conditions required by Sakharov are satisfied. The standard measure of the excess of baryons over antibaryon is the ratio [2, 3]

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-10} \quad (1)$$

where n_B is the number density of baryons, $n_{\bar{B}}$ is the number density of antibaryons and n_γ is the photon number density. However SM together with the Sakharov paradigm fails to predict order of magnitude because there is insufficient CP violation: for 3 generations of quarks the charge current can be written in terms of a single phase which leads to CP violation and is



characterised by a dimensionless parameter of order 10^{-20} [4]. This leads to a η which is too small by many orders of magnitude. Consequently new processes beyond SM are required. The suggestion that, within the framework of string theory, gravity with torsion can provide a new mechanism to generate more CP violation and leptogenesis has recently been suggested [5, 6, 7]. The gravitational background from string theory contains contributions from the graviton, described by a spin 2 field $g_{\mu\nu}$, an antisymmetric tensor field $B_{\mu\nu}$, the Kalb-Ramond field, and the dilaton, a scalar field ϕ . There is a simple gravitational background with a non-trivial Kalb-Ramond field, which has interesting phenomenology associated with leptogenesis. Because of gravitational universality, the mechanism could in principle generate baryogenesis directly but the detailed phenomenology involving sterile neutrinos is more compatible with experimental constraints.

The purpose of this paper is to present the ideas and analysis that are required to obtain the background solutions used in the model of leptogenesis proposed in [5]. We examine the equations of string cosmology for the gravitational sector in the presence of a central charge deficit and non-zero Kalb-Ramond field. Some explicit solutions will be considered for the equations which arise in lowest order in α' , the string tension. The backgrounds that result do not give a simple phenomenology. This is first rectified by considering the underlying conformal structure of the string effective action whereby a solution with constant torsion, valid to all orders in α' , can be found. The incorporation of fermions leads to a background solution with constant torsion even in lowest order in α' .

2. String theory effective action

The world sheet Polyakov string action classically has conformal invariance. The Polyakov action (in curved space-time) is given by

$$S = \frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G^{\mu\nu}(X) \quad (2)$$

where α' is inversely proportional to the string tension gives a partial differential equation for $G^{\mu\nu}(X)$. Classically, zeroth order in α this leads to a vanishing trace of the energy momentum tensor but quantum mechanically there is an anomaly; the requirement of the vanishing of this anomaly requires, at the quantum level, that $G^{\mu\nu}(X)$ satisfies a constraint in the form of a partial differential equation. However there is also a background antisymmetric field $B^{\mu\nu}(X)$ and a background dilaton field $\phi(X)$ ¹. Hence the Polyakov action has an additional contribution S_σ [8]

$$S_\sigma = \frac{1}{4\pi\alpha'} \left\{ \int_\Sigma d^2\sigma \sqrt{-\gamma} \phi(X) R^{(2)}(\sigma, \tau) + \int_\Sigma d^2\sigma B_{\mu\nu}(X) \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \right\} \quad (3)$$

where $d^2\sigma \equiv d\sigma d\tau$. However the presence of conformal anomalies modifies the action and consequently the equations of motion. On demanding conformal invariance at the quantum level, two additional equations are imposed which involve all the background functions. These constraints can be summarised through the construction of an effective Lagrangian; the Euler-Lagrange equations for this effective action are the constraints. Regarding the Polyakov action as a Bosonic two dimensional field theory, with coupling α' , the effective action can also be an expansion in α' . Solutions of the constraint equations will be called (infrared) fixed points (although the “points” are functions). Non-perturbative fixed points are those for which the effective action is considered to all orders in α' . Perturbative fixed points are obtained from the effective action in lowest order. Associated with $B_{\mu\nu}$ is a field strength $H_{\lambda\mu\nu} = \partial_{[\lambda} B_{\mu\nu]}$ where the square brackets around the indices denote antisymmetrization of the indices.

¹ It should be noted also that in theories with open strings the Kalb-Ramond field is the gauge field associated with string charge.

Fermions are usually coupled to the Einstein gravitational background through vierbeins e_d^μ (and inverse vierbeins e_μ^d) which characterise the local tangent space of the space-time manifold. We have

$$g_{\mu\nu}(x) = \eta_{ab} e_\mu^a(x) e_\nu^b(x), \quad e_a^\mu e_{b\mu} = \eta_{ab} \quad (4)$$

where η_{ab} is the Minkowski metric. Using vierbeins we can associate tensors with Latin indices to those with Greek indices e.g.

$$H_{d\mu\nu} = e_d^\lambda H_{\lambda\mu\nu}. \quad (5)$$

In the local Lorentzian frame given by the tetrad (whose components in the co-ordinate frame are given by the vierbeins) the action is the same as the flat one, provided that ordinary derivatives are replaced by covariant ones $\partial_a \rightarrow \bar{\nabla}_a$ (minimal coupling). This is obtained by requiring that $\bar{\nabla}_a \psi$ transforms under a boost of the tetrad according to the spinor and vector indices it carries [9]. The result that one finds in this way is the following

$$\bar{\nabla}_a \psi = e_a^\mu \left(\partial_\mu + \frac{i}{2} \bar{\omega}_{b\mu c} \Sigma^{bc} \right) \psi. \quad (6)$$

In the formula above $\Sigma^{ab} = \frac{i}{4} [\gamma^a, \gamma^b]$ is the generator of the Lorentz group representation on four-spinors, while $\omega_{a\mu b}$ is the Ricci rotation coefficient, defined as

$$\bar{\omega}_\mu^{ab} = e_\nu^a \bar{\nabla}_\mu e^{b\nu} = e_\nu^a \left(\partial_\mu e^{b\nu} + \bar{\Gamma}_{\mu\lambda}^\nu e^{b\lambda} \right). \quad (7)$$

where

$$\bar{\Gamma}_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda + e^{-2\phi} H_{\mu\nu}^\lambda \neq \bar{\Gamma}_{\nu\mu}^\lambda. \quad (8)$$

The justification for this new connection arises from the calculations of string amplitudes [10] to lowest order in α' ; hence the interactions of fermions with the background is given by the spin connection but now with this new connection [11].

In the string effective action this can be extended to include corrections [10, 12] at low orders in α' . Hence the fermion action is

$$S_{Dirac} = \frac{1}{2} \int d^4x \sqrt{-g} i \left(\bar{\psi} \gamma^a \bar{\nabla}_a \psi - \bar{\nabla}_a \bar{\psi} \gamma^a \psi + 2im \bar{\psi} \psi \right). \quad (9)$$

The second term is usually not written in flat space, as its contribution is equal to the first term plus a surface integral. However, the situation is different when spacetime is not flat. In fact the second term is needed in order to preserve unitarity, allowing for the cancellation of an anti-hermitean term involving the trace of the Ricci coefficients ω_{ac}^a .

We will show how to rewrite the spin connection as a background external axial vector. Using the gamma matrix algebra $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ and the definition $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, the Dirac action (9) can be rewritten in the following way

$$S_{Dirac} = \int d^4x \sqrt{-g} \bar{\psi} \left(i\gamma^a \partial_a + \hat{B}_d \gamma^5 \gamma^d - m \right) \psi \equiv S_{Dirac}^{\text{free}} + \int d^4x \sqrt{-g} \hat{B}_\mu J^{5\mu}, \quad J^{5\mu} \equiv \bar{\psi} \gamma^\mu \gamma^5 \psi \quad (10)$$

The axial vector \hat{B}^d is defined by

$$\hat{B}^d = \frac{1}{4} \varepsilon^{abcd} e_a^\mu \bar{\omega}_{b\mu c} \quad (11)$$

where in the last step we used (7) and the symmetry properties of the torsion-free Christoffel symbol $\Gamma_{\mu\nu}^\lambda = \Gamma_{\nu\mu}^\lambda$. (The axial vector \hat{B}^d can also be written as the dual of the torsion tensor

$$\hat{B}^d = -\frac{1}{4} \varepsilon^{abcd} e^{-2\phi} H_{abc} .) \quad (12)$$

Allowing for quantum fluctuations of the background, there will be an important additional contribution to \hat{B}^d from the fermion axial current [5]. In four space-time dimensions we can write

$$\hat{B}^\mu = \partial^\mu b, \quad (13)$$

where $b(x)$ is a pseudoscalar field (also termed the Kalb-Ramond (KR) axion field). The presence of the axial current will alter the perturbative fixed points qualitatively.

We construct actions in 4-dimensions; in flat space-time the requirement of conformal invariance requires space-time to have more dimensions, 10 is the critical value in the case of superstrings and 26 is the critical value in the case of bosonic strings. This makes the construction of phenomenologically relevant string theories challenging since these extra dimensions have to be compactified. Moreover for the low energy limit to contain SM, there need to be chiral fermions. This leads to the requirement that there are gauge fields in the extra dimensions. Candidates for the gauge group are, for example, $E_8 \times E_8$ and $SO(32)$. Much work has led, in the semiclassical limit of string theories, to 4 dimensional ground states with gauge fields, quarks and leptons with interactions governed by the gauge group $SU(3) \times SU(2) \times U(1)$ [13]. Progress has been made in the construction of models involving strings and branes with realistic phenomenology and would lead to a “top-down” approach.

However we will adopt an approach where we have non-trivial backgrounds described by a non-linear sigma model on the two-dimensional world sheet. The backgrounds are expectation values in coherent states of modes of the string corresponding to the graviton, Kalb-Ramond field and dilaton.

3. Fixed points in the absence of matter

Fixed points perturbative in α'

We will consider the equations that follow from the string effective action in lowest order in α' [8]; the resultant conformal invariance conditions are:

$$R_{\mu\nu} = \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g_{\mu\nu}^2 \nabla^2 \phi + \frac{1}{2} \exp(2\phi) [\partial_\mu b \partial_\nu b] \quad (14)$$

$$\nabla^2 b + 2 \partial_\lambda b \partial^\lambda \phi = 0. \quad (15)$$

The degrees of freedom of the extra dimensions are represented by a conformal field theory with central charge c_I ; we require that

$$c_I = 22 + \delta c \quad (16)$$

where δc is the central charge deficit (in a bosonic string theory). The remaining conformal invariance equation is

$$\delta c = -3 \exp(-\phi) \left\{ -R + \nabla^2 \phi + \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} \exp(2\phi) (\partial b)^2 \right\} \quad (17)$$

where for convenience α' is taken to be 1. In the Einstein frame, the metric is assumed to be in the Robertson-Walker form

$$ds^2 = -dt^2 + a(t)^2 \tilde{g}_{ij} dx^i dx^j, \quad (18)$$

where

$$\tilde{g}_{ij} dx^i dx^j = \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (19)$$

and θ and φ are the spherical polar angles. With this ansatz and, on defining the Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$, it can be shown that

$$\dot{b} = b_0 \frac{\exp(-2\phi)}{a(t)^3}, \quad b_0 = \text{const.} \quad (20)$$

and so \dot{b} is not a constant. Also

$$\delta c = 6 \exp(-\phi)(\dot{H} + 3H^2 + \frac{2k}{a(t)^2}). \quad (21)$$

However in the limit $t \rightarrow \infty$ these equations lead to

either (a): $H \rightarrow 0$, $\phi \sim \phi_0 + A \exp(-\beta t)$ where A and β are constants with $\beta > 0$, and $\dot{b} = b_0 \exp(-2\phi_0)$ and $k = \frac{1}{4}b_0^2 \exp(-2\phi_0) > 0$ (i.e. a closed Einstein universe). This case is not phenomenologically interesting.

or (b): $a(t) = t$ with asymptotically $H(t) \sim \frac{1}{t}$, $\phi = -2 \ln t + \phi_0$, and $\dot{b} = 2 \exp(-\phi_0) \sqrt{k} t$ (with $k \geq 0$). Moreover the four dimensional Ricci scalar $R = 6 \frac{(1+k)}{t^2}$ and $\delta c = 12 \exp(-\phi_0)(1+k)$ [8].

Neither case leads to a constant axial vector \widehat{B}^d (cf.(11)).

Fixed point non-perturbative in α'

In the absence of matter we have seen that the perturbative solution (b) of the conformal invariance conditions do not lead *asymptotically* to a constant torsion background. The effective theory to all orders in α' has been investigated sometime ago [14] using methods of conformal field theory. For a string theory (with four uncompactified dimensions) in non-trivial cosmological backgrounds, a world-sheet description is provided by a sigma model that can be identified with a conformal field theory of the Wess-Zumino-Witten type [6]:

$$L_{\text{conformal}} = -(\partial X^0)^2 - QX^0 R^{(2)} + \mathcal{L}_{\mathcal{WZW}}(O(3)) \quad (22)$$

where $\mathcal{L}_{\mathcal{WZW}}(O(3))$ is the Wess-Zumino-Witten Lagrangian over the $O(3)$ manifold. This group is relevant since it is assumed that the 3-spatial dimensions have been compactified into a 3-sphere: from a cosmological perspective we are interested in solutions which are isotropic and homogeneous. g_{ij} is the metric on the 3-sphere and H_{ijk} is the volume element [8]. This construction has led to exact solutions (due to a full implementation of conformal invariance) for cosmological bosonic backgrounds with non-trivial metric, antisymmetric tensor and dilaton fields. In the Einstein frame such a solution, consists of (i) a Robertson-Walker metric with a scale factor $a(t) \sim t$ where t is the cosmic time, (ii) a dilaton field ϕ that scales as $\phi(t) \sim -\ln a(t)$, and (ii) a KR axion field scaling linearly with the cosmic time, $\bar{b} \propto t$ with \bar{b} denoting the background value of b (cf. (13)). The resulting background axial vector $\widehat{\bar{B}}^d$ has only a non-trivial temporal component

$$\bar{b} \sim \text{const } t, \quad \partial^\mu \bar{b} \sim \epsilon^{\mu\nu\rho\sigma} e^{-2\phi} H_{\nu\rho\sigma}, \quad (23)$$

and

$$\widehat{\bar{B}}^0 \propto \dot{\bar{b}} = c \quad (24)$$

for constant c in the Robertson-Walker frame. This mechanism has a c independent of time and hence temperature in the expanding universe. In the absence of any arguments justifying phase transitions with decreasing temperature of the Universe which lead to lower magnitudes for c , it is necessary to choose c with a very small and phenomenologically acceptable value today. Such a choice of c would give negligible leptogenesis. However, in the presence of fermionic matter we will show that a perturbative fixed point can have a similar behaviour to (24). An important feature of the fermion induced fixed point is that it is possible to argue for a phase transition from a high to a low value of c .

4. Perturbative fixed point in the presence of fermionic matter

The antisymmetry of $H_{\mu\nu}^\lambda$ in its lower indices, implies that this field strength plays the rôle of a torsion tensor [15]. This suggests that this new connection (8) might be more fundamental than the Levi-Civita connection, and leads to different predictions whenever the Kalb-Ramond field is in a non-trivial configuration. In [6] a potential rôle of the H field for leptogenesis was emphasised but the effect of fermions on the fixed point was ignored. A detailed model for leptogenesis was presented in [5] (based on [7, 6]) where (a) the phenomenology of leptogenesis is discussed and (b) the issue of perturbative fixed points was raised.

We will now discuss the rôle of string matter in ensuring that torsion (provided by the antisymmetric tensor field strength) remains constant in time. Fermions are dealt with at the effective action level. In principle it is possible to take a “top-down” approach and start with a string theoretic model which contains the standard model particles and interactions and then deduce a low energy effective action. This is complicated and, moreover, no one string model has emerged as being the correct model. We shall consider perturbation theory in powers of α' (the Regge slope). In four-large-space-time dimensions, to lowest-order in α' , the Einstein-frame effective action including matter fields reads

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - 2\partial^\mu \phi \partial_\mu \phi - e^{-4\phi} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \frac{2}{3} \delta c \exp(2\phi) \right) + \int d^4x \sqrt{-g} e^{4\phi} \mathcal{L}_{\text{matter}}. \quad (25)$$

It is important to realise that $\mathcal{L}_{\text{matter}}$ contains the coupling of the antisymmetric tensor field strength $H_{\mu\nu\rho}$ with the axial fermionic current, J_μ^5 ; such terms do exist in our model as a consequence of the interpretation of the $e^{-2\phi} H_{\mu\nu\rho}$ in the Einstein term as contorsion (*cf.* (11)). $\mathcal{L}_{\text{matter}}$, to lowest order in α' , depends linearly on $-\frac{1}{4} \epsilon^{\mu\nu\rho\sigma} e^{-2\phi} H_{\mu\nu\rho} \bar{\psi} \gamma^\sigma \gamma^5 \psi$, where ψ are generic fermion fields. Such terms have not been considered explicitly in the literature before.

The corresponding equations of motion for the graviton, antisymmetric tensor fields and dilatons read, respectively:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= \kappa^2 \left(T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^H + T_{\mu\nu}^\phi + T_{\mu\nu}^c \right), \\ \nabla_\mu \left(e^{-4\phi} H^{\mu\nu\lambda} + \kappa^2 e^{2\phi} \epsilon^{\mu\nu\lambda\sigma} J_\sigma^5 \right) &= 0, \\ \square \phi - \frac{1}{3} e^{2\phi} \delta c + e^{-4\phi} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + 2\kappa^2 e^{4\phi} \mathcal{L}_{\text{matter}}^{\neq H} - \frac{1}{4} \kappa^2 e^{2\phi} \epsilon^{\mu\nu\lambda\sigma} H_{\mu\nu\lambda} J_\sigma^5 &= 0, \end{aligned} \quad (26)$$

where $J_\mu^5 = \bar{\psi} \gamma_\mu \gamma^5 \psi$ and $\mathcal{L}_{\text{matter}}^{\neq H}$ denotes the part of the matter lagrangian independent of couplings to $H_{\mu\nu\rho}$. $T_{\mu\nu}^i$ are the stress tensors pertaining to the various fields, $i = \text{matter}, H, \phi$ and c , with $T_{\mu\nu}^c = (\frac{1}{3} e^{2\phi} \delta c) g_{\mu\nu}$ the vacuum energy contribution. To these equations one should add the equations of motion for the matter fields, which have, however, important non-gravitational interactions, e.g. with gauge fields.

In four dimensions, in the *absence* of any matter fields, the equation for the antisymmetric tensor field is solved trivially by the replacement

$$H^{\mu\nu\lambda} = e^{4\phi} \eta^{\mu\nu\lambda\rho} \partial_\rho b, \quad (27)$$

(where b is the Kalb-Ramond pseudoscalar and $\eta^{\mu\nu\lambda\rho}$ is the scalar antisymmetric tensor ($\eta^{\mu\nu\rho\sigma} = \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}$). The Bianchi identity $\partial_{[\mu} H_{\mu\rho\lambda]} = 0$, where $[\dots]$ indicates antisymmetrization of indices, serves as a dynamical equation for the b field [14], an example of scalar-tensor duality. On contracting this identity with $\eta^{\mu\nu\rho\lambda}$, and, on noting that $\eta_{\mu\nu\rho\sigma} \eta^{\mu\nu\rho\lambda} = 6 \delta_\sigma^\lambda$, we obtain in the Einstein frame the following equation for b given by

$$\square b + 4\partial_\mu \phi \partial^\mu b = 0 \quad \Rightarrow \quad \ddot{b} + \left(3\frac{\dot{a}}{a} + 4\dot{\phi} \right) \dot{b} = 0. \quad (28)$$

Were we to require a solution with a time-independent \dot{b} ,

$$\dot{b} = K = \text{constant}, \quad (29)$$

then the dilaton field, in terms of the scale factor of the FRW Universe, $a(t)$, would be given by

$$\phi = -\frac{3}{4} \ln a \quad (30)$$

On using this expression for ϕ in the dilaton equation (26) we obtain

$$-\frac{3}{2} \left[\frac{1}{2} \ddot{a} + \left(\frac{\dot{a}}{a} \right)^2 \right] = \frac{1}{3} \frac{\delta c}{a^{3/2}} - 2 \frac{K^2}{a^3} + \dots \quad (31)$$

where the \dots denote contributions from matter fields with no explicit H -torsion terms.

We have also the Einstein equations for the metric field:

$$\begin{aligned} \left(\frac{\dot{a}}{a} \right)^2 &= \frac{1}{3} \kappa^2 \sum_i \rho_i, \\ \frac{\ddot{a}}{a} + \frac{1}{2} \left(\frac{\dot{a}}{a} \right)^2 &= -\frac{1}{2} \kappa^2 \sum_i p_i, \end{aligned} \quad (32)$$

where ρ_i and p_i indicate the energy density and pressure respectively of the various stress energy tensors, assumed to be ideal fluids.

In conjunction with (26) it is straightforward to see that there are no solutions compatible with a simple scaling of the scale factor, as in the radiation era -a conventional cosmology assumed in the leptogenesis scenario of [5].

However, in the presence of chiral fermions and gauge fields (such as the electromagnetic potential, colour gauge fields *etc*), it is known that the Bianchi identity for the torsion is anomalous and is modified [12], such that in the Einstein frame

$$\epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma} = \left(\epsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma} \right)_{;\mu} = \text{constant} \times F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} \equiv \text{constant} \times F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (33)$$

where the semicolon denotes covariant derivative with respect to the torsion-free connection. (N.B. the tensorial density, $\epsilon^{\mu\nu\rho\sigma}$, is covariantly constant; the chiral anomaly² and the anomaly in the Bianchi identity both arise from the same fermion loop involving chiral fermions.) The proportionality constant appearing on the right-hand-side depends on the details of the string/brane model. As mentioned in the text, Eq. (33) is also related to the axial anomaly [16].

The generation of Lorentz violating condensates of axial vector currents in strong coupling lattice gauge theories is shown to be possible in [17]. As the gauge coupling decreases from strong coupling, there will be a phase transition when the condensate will disappear. In string theoretic constructions of the standard model there is a relation between the string coupling g_s and the (grand unification) gauge coupling g_u typically of the form $g_u \propto g_s$ [18]. Since $g_s \sim \exp(\langle\phi\rangle)$, for solutions of $\langle\phi\rangle$ which tend to $-\infty$ as $t \rightarrow \infty$, g_u decreases below a critical value g_c as t increases. Moreover the expansion of the Universe also affects the ability to form condensates

² More generally the chiral anomaly equation is

$$\nabla_\mu J^{5\mu} \propto \frac{1}{32} \alpha' \left(R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right). \quad (34)$$

due to a decrease in the fermion density. Consequently the condensate can disappear in late eras of the Universe.

On assuming the existence of a condensate of gauge fields $\langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle$ ³ in the leptogenesis era, it is easy to see from (27) and (28) that the modified Bianchi identity reads

$$\ddot{b} + 3\frac{\dot{a}}{a}\dot{b} + 4\dot{\phi}\dot{b} \propto e^{-4\phi} \langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle \quad (35)$$

which implies that a *time independent solution* for \dot{b} is compatible with a constant dilaton⁴, as a perturbative fixed point;

To ensure a constant dilaton, and thus $\square\phi = 0$ in the dilaton equation of motion (26), one needs to impose

$$\frac{1}{4}\kappa^2 e^{2\phi} \epsilon^{\mu\nu\lambda\rho} H_{\mu\nu\lambda} \langle J_\rho^5 \rangle = e^{-4\phi} H_{\mu\nu\rho}^2 - \frac{1}{3} e^{2\phi} \delta c + \dots \quad (36)$$

where ... include contributions from the rest of the matter lagrangian. For consistency with rotational invariance only the temporal component $\langle J_0^5 \rangle \neq 0$ of the vacuum current condensate is assumed to be non-trivial. During the leptogenesis era we assume a time-independent condensate $\langle J_0^5 \rangle$ [17]. If the phase transition, when the condensate disappears, happens after that era, then the linear dependence of \dot{b} will cease. Also the condensate typically will vary with density [20] as a linear or higher power.

Substituting (36) in the original action (25), we obtain a potential term, of the form (the terms containing $\delta c e^{2\phi}$ cancel out):

$$S \ni \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \{-3 e^{-4\phi_0} H_{\mu\nu\rho}^2 + \dots\} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \{-18 e^{4\phi_0} (\dot{b})^2 + \dots\} \quad (37)$$

where ϕ_0 is the constant dilaton value. The constant \dot{b} is fixed to produce leptogenesis, as discussed in the text.

However, the vacuum energy induced by the Kalb-Ramond axion field in the form of its kinetic energy $(\dot{b})^2 e^{4\phi_0}$ is too large for the MeV values assumed for \dot{b} (necessary to produce sufficient leptogenesis) [5]; so the vacuum energy needs to be cancelled. Judicious choices may be made in the vacuum energy contributions e.g. with a negative δc (i.e. a central charge deficit). Another possibility is anti-de Sitter contributions that have been found in [21] within the framework of a Randall-Sundrum brane world in a string theory model. Consequently with fine-tuning one may achieve a constant antisymmetric-tensor-background induced torsion during radiation era consistent with small vacuum energy as required by standard cosmology. In later eras, due to the effects of the dilaton, there is a decay in time of the vacuum energy of the torsion and also of the cancelling vacuum contributions. Moreover, as mentioned earlier, the non-conservation of the fermion axial current (whose divergence is proportional to $F_{\mu\nu} \tilde{F}^{\mu\nu}$ as a result of the chiral anomaly equation (34)), is compatible with the existence of a condensate and with Eq. (35).

5. Conclusions

In the presence of fermions we have shown that, in four dimensions, the constant torsion backgrounds can arise even when the string effective action is treated in lowest order in α' . The backgrounds that we have discussed break both CP and CPT invariance when coupled to fermions. This leads to a model of leptogenesis [5]. The role of string theory is crucial in giving backgrounds with torsion. The torsion based approach opens up interesting new models for leptogenesis.

³ A D-particle based mechanism for the formation of such condensates is given in [5].

⁴ Typically the behaviour of the dilaton can be parametrised [19] as $\phi = \Phi_0 \ln[a(t)]$ with Φ_0 negative. ϕ is a slowly varying function of time which has to decay away before the epoch of Big-Bang-nucleosynthesis, so as not to disturb its delicate conditions, and so in the era of leptogenesis (that precedes nucleosynthesis in our model) we will for simplicity take ϕ to be a constant $\phi = \Phi_0$.

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References

- [1] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5**, 32 (1967) [JETP Lett. **5**, 24 (1967)] [Sov. Phys. Usp. **34**, 392 (1991)] [Usp. Fiz. Nauk **161**, 61 (1991)].
- [2] D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **148**, 175 (2003) [astro-ph/0302209].
- [3] P. A. R. Ade *et al.* [Planck Collaboration], Astron. Astrophys. **571**, A16 (2014) [arXiv:1303.5076 [astro-ph.CO]].
- [4] R. N. Mohapatra and P. B. Pal, World Sci. Lect. Notes Phys. **60**, 1 (1998) [World Sci. Lect. Notes Phys. **72**, 1 (2004)].
- [5] M. de Cesare, N. E. Mavromatos and S. Sarkar, arXiv:1412.7077 [hep-ph];
- [6] J. Ellis, N. E. Mavromatos and S. Sarkar, Phys. Lett. B **725** (2013) 407 [arXiv:1304.5433 [gr-qc]];
- [7] N. E. Mavromatos and S. Sarkar, Eur. Phys. J. C **73** (2013) 3, 2359 [arXiv:1211.0968 [hep-ph]].
- [8] N. E. Mavromatos, Lect. Notes Phys. **592** (2002) 392 [hep-th/0111275].
- [9] T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961); D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964) [Erratum-ibid. **36**, 1103 (1964)].
- [10] D. J. Gross and J. H. Sloan, Nucl. Phys. B **291**, 41 (1987);
R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B **293**, 385 (1987); M. C. Bento and N. E. Mavromatos, Phys. Lett. B **190**, 105 (1987).
- [11] J. Scherk and J. H. Schwarz, Phys. Lett. B **52**, 347 (1974).
- [12] M. J. Duncan, N. Kaloper and K. A. Olive, Nucl. Phys. B **387**, 215 (1992).
- [13] M. Dine, "Introduction To String Theory In Four-dimensions.", in "Dine, M. (Ed.): String Theory in Four Dimensions" 1-40; B. Zwiebach, "A first course in string theory," Cambridge, UK: Univ. Pr. (2009) 673 p
- [14] I. Antoniadis, C. Bachas, J. R. Ellis and D. V. Nanopoulos, Nucl. Phys. B **328**, 117 (1989).
- [15] F. W. Hehl, P. Von Der Heyde, G. D. Kerlick and J. M. Nester, Rev. Mod. Phys. **48**, 393 (1976);
I. L. Shapiro, Phys. Rept. **357**, 113 (2002) [hep-th/0103093] and references therein.
- [16] See, for instance: S. Weinberg, *The Quantum Theory of Fields. Volume II: Modern Applications*. (Cambridge University Press 2001) ISBN 0-521-55002-5. ; J. P. Blaizot and E. Iancu, *QCD perspectives on hot and dense matter. Proceedings*, NATO Advanced Study Institute, Summer School, Cargese, France, August 6-18, 2001 (NATO science series II: Mathematics, physics and chemistry. 87).
- [17] E. T. Tomboulis, Phys. Rev. D **85** (2012) 024042 [arXiv:1105.3720 [hep-th]].
- [18] B. Dundee, S. Raby and A. Wingerter, Phys. Rev. D **78** (2008) 066006 [arXiv:0805.4186 [hep-th]].
- [19] A. B. Lahanas, Phys. Rev. D **83**, 103523 (2011) [arXiv:1102.4277 [hep-ph]]; C. M. Bender, N. E. Mavromatos and S. Sarkar, Phys. Rev. D **87** (2013) 5, 055021 [arXiv:1212.6715 [hep-ph]].
- [20] S. Goda and D. Jido, Phys. Rev. C **88** (2013) 6, 065204 [arXiv:1308.2660 [nucl-th]].
- [21] N. E. Mavromatos and J. Rizos, Int. J. Mod. Phys. A **18**, 57 (2003) [hep-th/0205299]; Phys. Rev. D **62**, 124004 (2000) [hep-th/0008074].